(8 pages) **Reg. No. : ....**

Code No. : 6374 **Sub. Code : ZMAE 21**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics

Elective — CLASSICAL MECHANICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The equation of motion is a differential equation of order

(a) 3 (b) 0

(c) 1 (d) 2

2. If there exist holonomic constrains expressed in 5 equations, then the system of 50 particles has ———— degrees of freedom.

(a) 55 (b) 45

(c) 145 (d) 150

3. The Lagrangian L is defined as

(a) $T + V$

(b) $T - V$

(c) $V - T$

(d) $T.V$

4. The equation $\sum_i F_i^{(a)} \cdot \delta r_i = 0$ is called

(a) the principle of virtual work

(b) the equation of motion

(c) the Lagrangian equation

(d) the Maxwell equation

5. The total length of any curve going between points 1 and 2 is

(a) $\int_{x_1}^{x_2}\left(1+\left(\frac{dy}{dx}\right)^2\right)dx$

(b) $\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

(c) $\int_{x_1}^{x_2}\left(1+\left(\frac{dy}{dx}\right)^{1/2}\right)dx$

(d) $\int_{x_1}^{x_2}\sqrt{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2}\,dx$

6. For any parametric family of curves $J(\alpha)=\int_{x_1}^{x_2} f\left(y(x,\alpha),\dot{y}(x,\alpha),x\right)dx$ the condition for obtaining a stationary point is

(a) $\left(\frac{dJ}{dx}\right)_{\alpha=0}\Big| = 0$ (b) $\left(\frac{dJ}{d\alpha}\right)_{\alpha=0}\Big| = 0$

(c) $\left(\frac{dJ}{d\alpha}\right)_{x=0}\Big| = 0$ (d) $\left(\frac{dJ}{d\alpha}\right)_{y=0}\Big| = 0$

7. The areal velocity is

(a) $n^2\dot{\theta}$ (b) $\frac{1}{2}r^2\theta$

(c) $\frac{1}{2}r^2\dot{\theta}$ (d) $r^2\dot{\theta}$

8. Consider a plot of V' against r for the specific case of an attractive inverse square law of force $f=-\frac{k}{r^2}$. The potential energy for this force is

(a) $V=\frac{k}{r}$ (b) $V=\frac{k}{r^2}$

(c) $V=-\frac{k}{r}$ (d) $V=0$

9. The eccentric anomaly ψ is defined by the relation

(a) $r=a\left(1+e\cos\psi\right)$ (b) $r=a\left(1-e\cos\psi\right)$

(c) $r=a\left(1-\cos\psi\right)$ (d) $\psi=\frac{ae}{r^2}$

10. The Kepler's equation is

(a) $\omega t=\psi-e\sin\psi$ (b) $\omega t=\psi+e\sin\psi$

(c) $\omega t=\psi-e\cos\psi$ (d) $\omega t=\psi+e\cos\psi$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the conservation theorem for the angular momentum of a particle.

Or

(b) Define a rigid body and show that in a rigid body the internal forces do no work. What can you say about the internal potential of such body.

12. (a) Obtain the Lagrange equations of motion for a spherical pendulum.

Or

(b) Derive the Lagrange equation of motion of a bead sliding on a uniformly rotating wire in a force-free space.

13. (a) Show that the shortest distance between two points in a plane is a straight line.

Or

(b) Explain Braschistochrone problem.

14. (a) Show that the central force motion of two bodies about their center of mass can always be reduced to an equivalent one-body problem.

Or

(b) Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$

15. (a) Obtain the differential equation for the orbit if the force law f is known.

Or

(b) Prove that for the Kepler problem there exists a conserved vector A defined by $\overline{A} = \overline{P} \times \overline{L} - m\,k\frac{\overline{r}}{r}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove the conservation theorem for the linear momentum of a system of particles.

Or

(b) (i) Explain holonomic and nonholonomic constraints with suitable examples.

(ii) State the two types of difficulties due to constraints in the solution of mechanical problems.

17. (a) Derive Lagrange's equation of motion from D'Alembert's principle.

Or

(b) Show that the kinetic energy of a system can always be written as the sum of three homogeneous functions of the generalized velocities.

18. (a) Derive the Euler-Lagrange differential equations.

Or

(b) Derive Lagrange's equations for non holonomic systems.

19. (a) State and prove Kepler's second law of planetary motion.

Or

(b) A particle moves in a central force field given by the potential $V = -k\frac{e^{-ar}}{r}$ where k and a are positive constant. Using the method of the equivalent one-dimensional potential discuss the nature of the motion.

20. (a) Obtain the equation of motion for the particle moving under the influence of a central force $f = -k/r^2$.

Or

(b) (i) For the Kepler's equation $\omega t = \psi - e\sin\psi$ prove that $\tan\theta/2 = \sqrt{\frac{1+e}{1-e}}\tan\psi/2$

(ii) Derive the orbit equation for the Kepler problem using Laplace–Runge–Lenz vector.